Measuring Markups with Revenue Data

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Abstract

When output prices are unobserved, standard production-based markup estimators are biased and inconsistent because they are unable to distinguish whether firms have higher revenues due to higher prices or higher quantities. Building on work designed for competitive environments, we propose a novel method that solves this problem using only revenue data. We flexibly model markups as a specified function of observables and fixed effects, supporting a broad class of variable-markup frameworks. We explicitly adopt a Markovian revenue productivity process, a commonly implicit assumption in the literature. Our suggested two-step approach is simple in concept and implementation, requiring only common regression techniques.

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The composition of firms in modern economies is changing: average firm size is expanding, industries are concentrating, and business dynamism is declining (Grullon et al., 2019; Decker et al., 2016). Researchers have proposed several explanations for the driving forces behind these secular trends. One set of hypotheses attributes these trends to technological change or intangible capital, typically with positive welfare implications for consumers (Autor et al., 2020; Bessen, 2020). Another set attributes them to market power or entry barriers, typically with negative welfare implications for consumers (De Loecker et al., 2020; Gutiérrez and Philippon, 2019). While both perspectives may be partly true, their policy responses differ dramatically. Central to this discourse is the firm-level markup—the output price to marginal cost ratio—a key indicator of market power. The size of these trends and their potential welfare implications calls for economists to reliably measure such markups consistently across industries and over time.

Recent studies respond to this call by adopting the production approach to markup estimation on large firm-level datasets to draw conclusions for entire industries or economies (De Loecker et al., 2020; Traina, 2018; Diez et al., 2018; Calligaris et al., 2018). However, econometricians originally designed the production-function estimators underpinning this approach for perfectly competitive environments, where variation in prices can only reflect variation in quality. In imperfectly competitive environments, firm pricing confounds the link between revenues and quantities, indicating a fundamental role of detailed price information to recover unbiased and consistent markup estimates (Klette and Griliches, 1996; Doraszelski and Jaumandreu, 2019; Bond et al., 2021).[1] Yet, most production datasets lack such information, offering only revenue data.[2] Consequently, researchers interested in studying competition face a significant gap between the theoretical models and the available data.

[1] Using an industry price index to recover quantities does not solve this problem (Bond et al., 2021) except in two special cases: perfect competition, or no firm heterogeneity in prices (Hashemi et al., 2022).

[2] Compustat, Worldscope, and Orbis financial-statement datasets, as well as the US, Colombian, Chilean, India, Indonesia, and Slovenian manufacturing surveys, all do not include output price information.

To bridge this gap, we present an approach that delivers unbiased and consistent estimates using only revenue data. Starting from the firm's cost-minimization problem, we specify an input share regression that links observable variables to unobservable firm's profitability. Drawing on insights from the production function literature, we introduce a control function for markups that allows us to recover estimates of revenue elasticities and unexpected productivity shocks. The control function flexibly models markups as a specified function of observables and fixed effects, proxying for determinants of market power such as demand and market conditions, supporting a broad class of variable-markup frameworks. In the absence of price data, we show controlling for markups instead solves the omitted price bias generated using only revenues. We then leverage the stochastic process of productivity to estimate the production function, thus recovering markups. We depart from modeling the physical productivity process and instead model the revenue productivity process. This approach underlies many study using revenue data (Klette and Griliches, 1996; De Loecker, 2011; De Loecker and Warzynski, 2012; De Loecker et al., 2020), and is consistent with persistence results in Foster et al. (2008). Given this minimal setup, our two-step estimator is straightforward, consistent, and easily applied to readily accessible datasets.

Our key contribution is to offer a novel method to infer markups without requiring information on output prices, thus solving a critical long-standing problem in estimating production functions: the omitted price bias. The first stage of our suggested estimator is a share regression in the spirit of Gandhi, Navarro, and Rivers (2020), which directly estimates the revenue elasticity using the firm's cost minimization first-order condition. The second stage then uses information and timing assumptions to separate the effects of inputs on prices and output, thus recovering its output elasticities. We can then measure unbiased and consistent firm-level markups with these estimates. Our framework offers modifications to traditional production function estimators (Blundell and Bond, 2000) that apply more restrictive productivity structures. The strength of our approach is that it applies to a wide range of data and market settings, including potential applications in macroeconomic, trade, labor, and finance studies. Our approach might outperform standard estimators even when price data are available. For example, even if we had information on output prices, we would need to quality-adjust physical quantities to make sure outputs are comparable across firms, yet quality data are very rare in practice. Many firms also produce multiple products, which complicates how researchers should interpret quantity data without added assumptions on the production process De Loecker (2011). Our approach eases both concerns and is thus attractive even with price data.

The literature on production function estimation has long noted the omitted price bias in measuring markups with revenue data. Klette and Griliches (1996) shows that markup estimators are biased and inconsistent without information on output prices. This bias arises because unobserved prices enter the residual of the estimating equation but also affect firm input choices (e.g., higher prices induce the firm to use more inputs to increase output). Consequently, omitted prices cause a classic simultaneity problem. Doraszelski and Jaumandreu (2019) extends these results to modern production function estimators, while Bond et al. (2021) further shows that the bias should lead to unitary markups for profit-maximizing firms. We see our method as generalizing the solution in Klette and Griliches (1996), which relies on the restrictive assumptions of a constant elasticity demand system and monopolistic competition, thus constant and homogeneous markups. Beyond typical parametric critiques, these assumptions narrow the very concept of competition and imply zero markup variation within industries. More recently, De Loecker et al. (2016) uses observed output prices to control for unobserved input price biases, but the authors work in a rare setting where output price data are available. Foster, Haltiwanger, and Syverson (2008) suggests that the omitted price bias is so significant that it changes the observed correlation between physical productivity and revenue productivity. De Ridder et al. (2021) shows although we cannot recover precise markup levels from revenue data, we can estimate their dispersion both in the cross-section and over time. In sum, the literature has shown that the omitted price bias is important, but has not yet offered a general solution for resolving it. We contribute to this literature by offering a promising avenue to study market power in a wide range of settings by tailoring the productivity process and proxy variable assumptions.

Besides addressing the omitted price bias, our paper also relates to the literature on transmission bias: firms optimally choose their inputs as a function of their productivity, so productivity simultaneously determines both output and inputs (Marschak and Andrews, 1944). In an empirical analysis, regressing output on inputs fails to identify a firm's production function because the unobservable productivity term transmits into the input decisions, thus creating a critical endogeneity problem. The literature overcomes this bias with assumptions about a firm's production or productivity process. Broadly, proxy variable estimators (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015; Gandhi et al., 2020) assume that observable inputs can control for unobserved productivity, while dynamic panel estimators (Blundell and Bond, 2000) parametrically impose linearity in the productivity process. Our suggested method is in the tradition of proxy variable estimators meaning that it can allow for flexible productivity process, but it allows researchers to relax the critical assumption of competitive output markets. Our work also relates to earlier attempts at forming control functions for revenue productivity directly, as in Flynn, Traina, and Gandhi (2019). One appealing feature is that in general models of competition, higher planned markups induce lower chosen flexible inputs; this suggests a straightforward way to microfound the markup control function in an input demand equation, as in Olley and Pakes 1996. We also derive results showing how to extend methods in the tradition of dynamic panel estimators.

Our main estimator builds on this line of work by addressing omitted price and transmission bias simultaneously. We do so by specifying a control function for markups, as opposed to the typical control function for physical productivity. We then directly estimate the firm's revenue elasticity by regressing its flexible input's log cost share of revenue on log inputs and fixed effects. Though similar in spirit to the first stage of Gandhi, Navarro, and Rivers (2020), we do not identify the flexible input's output elasticity from this share regression.

Instead, we use it to partially identify the elasticity by identifying a combination of the elasticity and markup. We then substitute this composite into the revenue production equation to control for unobserved revenue productivity. Together, this method allows us to identify physical output elasticities relying only on revenue data.

Road-Map. Section 1 introduces a general production model with imperfect competition in product markets. Section 2 describes how to adapt the production model for imperfectly competitive environments, and how to use our proposed method to recover markups. Section 3 compares our estimator to existing production function estimation methods and models of imperfect competition. Section 4 concludes.

1 A Production Model with Markups

In this section, we present a structural model consistent with widely used approaches to firm-level production function and markup estimation. We consider the contemporary case of imperfect competition in the output market and cost minimization for a flexible input. This setup links the firm's markup to the flexible input's output elasticity and cost share of revenue (Hall, 1988; Basu and Fernald, 2002; Petrin and Sivadasan, 2013; De Loecker and Warzynski, 2012). After deriving the link in our model, we discuss identification and estimation problems when researchers only have data on revenues (not quantities). Throughout, we use uppercase to refer to levels of variables and lowercase to refer to logs of variables.

1.1 Production Technology and Productivity

The economy is populated by profit-maximizing firms with idiosyncratic productivity. Each firm generates final output combining a competitively supplied flexible input X_{it}, and a vector of nonflexible inputs, K_{it}, according to the following function:

$$Q_{it} = \Omega_{it} F(X_{it}, K_{it}) \tag{1}$$

with Ω_{it} representing firm i's physical productivity (TFPQ in the parlance of Foster et al. (2008)), and $F(\cdot)$ being the firm's production function.[3] X_{it} is flexible in the sense that it is both variable and static: firms may adjust it in each period after observing the realization of state variables such as productivity, and its choice has no dynamic implications. K_{it} is fixed, dynamic, or both. For exposition, we refer to X_{it} as intermediates, K_{it} as capital, and assume capital is fixed so that it may not respond to current period state variables. In practice, the flexibility of X_{it} is crucial to derive a markup formula that can be estimate on firm-level data.

As in Olley and Pakes (1996), the log productivity term, $a_{it} = \log(\Omega_{it})$, is additively separable into a part known to the firm when making input decision, $\omega_{it} \in \mathcal{I}_{it}$, and an i.i.d. error term, $\varepsilon_{it} \notin \mathcal{I}_{it}$. Firm production is thus

$$q_{it} = f(x_{it}, k_{it}) + \omega_{it} + \varepsilon_{it} \tag{2}$$

1.2 Residual Demand and Pricing

Firms may have market power in the output market, so that a single firm's residual demand curve may not be perfectly elastic and takes the following form,

$$q_{it} = \psi_{it} p_{it} + \kappa_{it} \tag{3}$$

With ψ_{it} being the residual demand elasticity that firms internalize in their pricing choices and κ_{it} demand shifter that does not affect markup decisions. The residual demand curve accounts for competitor responses and thus is a mix of demand and conduct (Baker and Bresnahan, 1988). For example, a

[3]The production function is concave and diffentiable at every point. We omit the time subscript for sake of exposition, but in principle the production function can vary over time.

monopolist's residual demand curve is the market demand curve. In a perfectly competitive market, a firm's residual demand curve is flat because any change in its output is offset by a change in its competitors' output. The same reasoning applies to a Bertrand duopolist with undifferentiated products. It includes information on own and cross elasticities as well as on other firms' decisions, and it defines an optimal markup level that profit-maximizing firms endogenously set. In this sense, without imposing additional structure on the source of market power, ψ can be a mix of competition and demand factors.

Firms choose a price and quantity pair $\{P_{it}, Q_{it}\}$ along their residual demand curves, subject to technology constraints. Firms may choose to price above marginal cost, thus generating a markup. Markups are equilibrium objects that depend on demand, costs, market structure, and possibly other factors. As common to the markups estimated from static cost optimization, the approach we develop here is consistent with static pricing only and is therefore inconsistent with sticky price and customer capital models, though similar principles could be applied. We are otherwise fairly agnostic about the source of markups, which could arise from product differentiation, consumer tastes, technological advantages, or concentrated markets, among others.

1.3 Optimization, Information, and Timing

We assume that firms minimize costs and we study the per-period static cost minimization problem. Each firm uses expected output in its minimization problem because it knows that it must account for an as-yet-unknown portion of productivity, ε_{it}.

The timing of the problem is as follows. At the beginning of the period, firms know their capital stock, K_{it}, and a part of their contemporaneous productivity, ω_{it}. First, each firm plans an optimal markup, μ_{it}, relying on the expectation about total factor productivity, a_{it}, along with other information observable to the firm regarding its residual demand curve. Second, they choose the corresponding intermediate inputs, X_{it}, to implement these plans given their residual demand curve, technology, capital stock, and expected productivity. Finally, productivity is fully realized, production occurs, and markups are realized. Markups are generally not orthogonal to productivity, since the firm uses its expectation about productivity to plan a markup.

Given this setup, the firm's cost minimization problem with time t information, $\mathcal{I}_{it}$, is

$$\begin{aligned} \min_{X_{it}} \quad & C_{it} X_{it} \\ \text{s.t.} \quad & Q_{it} = \mathbb{E}[A_{it}|\mathcal{I}_{it}] F(X_{it}, K_{it}) \end{aligned}$$

with C_{it} being the cost of the flexible inputs. As we will discuss in the estimation section, this will be crucial for estimating markups and is potentially firm-specific. The Lagrangian of the cost minimization is

$$\mathcal{L}_{it} = C_{it} X_{it} + \Lambda_{it} (Q_{it} - \mathbb{E}[A_{it}|\mathcal{I}_{it}] F(X_{it}, K_{it})$$

with Λ_{it} being the Lagrangian multiplier, thus the cost of relaxing the quantity constraint. In this respect, Λ_{it} represents firm i's marginal cost. The first-order condition for the flexible input X_{it} is

$$[X_{it}] : \quad C_{it} = \Lambda_{it} \mathbb{E}[A_{it}|\mathcal{I}_{it}] \frac{\partial F(X_{it}, K_{it})}{\partial X_{it}}$$

Denote $\mathcal{M}_{it}$ as the markup, P_{it} as output price, we can manipulate the first-order condition to yield

$$
\begin{aligned}
\mathcal{M}_{it} &= \frac{P_{it}}{\Lambda_{it}} \\
&= \mathbb{E}[A_{it}|\mathcal{I}_{it}]\frac{\partial F(X_{it}, K_{it})}{\partial X_{it}}\frac{P_{it}}{C_{it}} \\
&= \frac{\mathbb{E}[A_{it}|\mathcal{I}_{it}]}{A_{it}}\frac{\partial F(X_{it}, K_{it})}{\partial X_{it}}\frac{A_{it}X_{it}}{Q_{it}}\frac{P_{it}Q_{it}}{C_{it}X_{it}} \\
&= \frac{\mathbb{E}[A_{it}|\mathcal{I}_{it}]}{A_{it}}\frac{\partial F(X_{it}, K_{it})}{\partial X_{it}}\frac{X_{it}}{F(X_{it}, K_{it})}\frac{P_{it}Q_{it}}{C_{it}X_{it}} \\
&= \frac{\mathbb{E}[\mathcal{E}_{it}]}{\mathcal{E}_{it}} f_{it}^X \frac{R_{it}}{C_{it}X_{it}}
\end{aligned}
$$

where the last line comes from the fact that the firm knows part of its productivity before its input choice, $\frac{\mathbb{E}[A_{it}|\mathcal{I}_{it}]}{A_{it}} = \frac{\Omega_{it}\mathbb{E}[\mathcal{E}_{it}]}{\Omega_{it}\mathcal{E}_{it}} = \frac{\mathbb{E}[\mathcal{E}_{it}]}{\mathcal{E}_{it}}$, the definition of the output elasticity, $f_{it}^X = \frac{\partial F(X_{it}, K_{it})}{\partial X_{it}}\frac{X_{it}}{F(X_{it}, K_{it})}$, and revenues, $R_{it} = P_{it}Q_{it}$.

Defining now the expectation term as $b_{it} = \log \mathbb{E}[\mathcal{E}_{it}]$, and writing this result in logs gives us

$$
\mu_{it} = \log f_{it}^X + r_{it} - c_{it}x_{it} + b_{it} - \varepsilon_{it} \tag{4}
$$

That is, the log markup, μ_{it}, is the sum of the intermediate input's output elasticity, f_{it}^X, the intermediate input's log inverse cost share of revenue, $r_{it} - c_{it}x_{it}$, and the unplanned productivity term, $b_{it} - \varepsilon_{it}$. Equation 4 is the key relation that we use to estimate markups with our method.

2 Measuring Markups with Revenue Data

In this section, we describe our novel approach to estimate markups with revenue data and discuss the underlying assumptions. The aim is to identify physical output elasticities (and error terms) to measure markups from the first-order condition of the per-period static cost minimization problem.

We assume we observe data for a panel of firms $i = 1, 2, ..., N$ over periods

$t = 1, 2, \ldots, T$. We let the data take a short panel form: the number of firms grows large for a fixed T. For each firm, we observe revenue $R_{it} = P_{it}Q_{it}$, expenditures on a competitively supplied flexible input X_{it} with cost C_{it}, and a vector of nonflexible inputs K_{it}. In practice, the choice of X_{it} might be energy, materials, labor, or some combination thereof. We recommend choosing the input that is most likely to satisfy the competitively supplied and flexible assumptions based on the empirical setting.

2.1 Problems with Revenue Data

Recovering markups from equation 4 requires an estimate of the output elasticity f_{it}^X. However, we cannot simply regress revenue on inputs to get this estimate for two critical reasons. The first reason is the omitted price bias emphasized in Klette and Griliches (1996): higher markups induce the firm to decrease input use, which increases prices and thus (all else equal) revenues. The second reason is the transmission bias emphasized in Marschak and Andrews (1944): higher physical productivity induces the firm to increase input use, which increases output and thus (all else equal) revenues.

One approach in the literature is to ignore the distinction between revenues and quantities, and estimating firm production function using common estimation techniques specifically designed to address the transmission bias. Specifically, the control function approach pioneered by Olley and Pakes (1996) and the dynamic panel methods proposed by Blundell and Bond (2000) are the most common methods. However, these existing methods to estimate production function parameters are designed for perfectly competitive environments and rely on physical quantity data. An empirical strategy that treats the revenue production function estimates as though they are physical production function estimates confounds demand with productivity (Foster et al., 2008) and results in markup estimates devoid of empirical content (Bond et al., 2021). Hence, we cannot use them in this context. Our method tackles this issue directly by including a control function approach in the spirit Olley and Pakes (1996) and allowing for imperfect competition. It requires only revenue data and is a generalization of the Klette and Griliches (1996)'s correction.

2.2 Our Approach: Controlling for Markups

At the core of our method there is the law of motion of productivity and a control function to proxy for markups. The former is common to the literature both for control function approach, that assumes a Markov process for total factor productivity, and for dynamic panel methods, that assumes a linear process. Given that our aim is to provide an estimation method using revenue data, we focus instead on revenue productivity. The latter relates to the approach used in the control function literature and brings insights also from the share regression proposed by Gandhi et al. (2020). Given that our approach deals with markup estimation using only revenue data, we propose to control for markups directly rather than only for unobserved productivity. In what follow, we describe the approach in details.

Assumption 1 (The Stochastic Process of Revenue Productivity) *Let revenue productivity $\nu_{it} = p_{it} + \omega_{it}$ (the sum of prices p_{it} and physical productivity ω_{it}) follow a Markov process with additively separable mean-zero shocks η_{it}. Hence, $\mathcal{P}(\nu_{it}|\mathcal{I}_{it-1}) = \mathcal{P}(\nu_{it}|\nu_{it-1})$, and $\mathbb{E}[\eta_{it}|\mathcal{I}_{it-1}] = 0$.*

Following Assumption 1, we can write the stochastic process of revenues productivity as

$$\nu_{it} = g(\nu_{it-1}) + \eta_{it} \tag{5}$$

for some continuous function $g(\nu_{it-1}) = \mathbb{E}[\nu_{it}|\nu_{it-1}]$. This stochastic process is consistent with the persistence in revenue productivity shown in Foster et al. (2008) and, relative to the literature, it is equivalent to the existing assumptions of Klette and Griliches (1996), De Loecker (2011), and De Loecker and Warzynski (2012). De facto, this is implicitly assumed in every study using revenue data to estimate a production function (e.g., in De Loecker et al., 2020).

Now, we write firm revenues as

$$
\begin{aligned}
r_{it} &= p_{it} + q_{it} \\
&= p_{it} + f(x_{it}, k_{it}) + \omega_{it} + \varepsilon_{it} \\
&= f(x_{it}, k_{it}) + \nu_{it} + \varepsilon_{it}
\end{aligned}
$$

Where the second line comes from the definition of $f(x_{it}, k_{it})$, and the third line comes from the definition of the revenue productivity ν_{it}. Therefore, we can express the revenue production function as

$$
r_{it} = f(x_{it}, k_{it}) + g(\nu_{it-1}) + \eta_{it} + \varepsilon_{it} \tag{6}
$$

The latter can be used to estimate production function parameters leveraging assumptions on firms' information and timing to construct moment conditions. However, we first need to recover a measure of revenue productivity to operationalize it. To do so, we use our structural framework. Specifically, we interpret the first-order condition 4 as a markup function and use it to identify revenue elasticities.

Assumption 2 (The Markup Control Function.) *Let markups be a function of inputs, firm and time fixed effects ι_i and τ_t, and a vector of other firm-time varying observables relevant in determining markups $\mathbf{D}_{it}$*

$$
\mu_{it} = h(x_{it}, k_{it}, \iota_i, \tau_t, \mathbf{D}_{it}) \tag{7}
$$

We return to examples of possible observables in Section 3. One appealing feature is that in general models of competition, higher planned markups induce lower chosen intermediates; this suggests a straightforward way to microfound the markup control function in an input demand equation, as in Olley and Pakes (1996). Rewrite the first-order condition 4 as

$$
\begin{aligned}
c_{it}x_{it} - r_{it} =& \log f^X_{it} - \mu_{it} + b_{it} - \varepsilon_{it} && (8)\\
=& \log f^X_{it} - h(x_{it}, k_{it}, \iota_i, \tau_t, \mathbf{D}_{it}) + b_{it} - \varepsilon_{it} && (9)
\end{aligned}
$$

The left-hand side is the intermediates log cost share of revenue. The term $\log f^X_{it} - \mu_{it}$ on the right-hand side is the log revenue elasticity with respect to input x_{it}, a mix of supply and demand parameters.[4] As productivity is Hicks-neutral, the log elasticity term f^X_{it} is a function of inputs only: $f^X_{it} = f^X(x_{it}, k_{it})$. Combining the revenue elasticity terms into a single function $s(x_{it}, k_{it}, \iota_i, \tau_t, \mathbf{D}_{it}) = \log f^X(x_{it}, k_{it}) - h(x_{it}, k_{it}, \iota_i, \tau_t, \mathbf{D}_{it})$, our first stage estimating equation becomes:

$$c_{it}x_{it} - r_{it} = s(x_{it}, k_{it}, \iota_i, \tau_t, \mathbf{D}_{it}) + b_{it} - \varepsilon_{it} \tag{10}$$

To operationalize this equation, a researcher may (nonparametrically) regress the intermediates share of revenues on inputs, fixed effects, and a vector of markup determinants $\mathbf{D}_{it}$ to get an estimate of the revenue elasticity $\hat{s}_{it} = \widehat{\log f^X_{it} - \mu_{it}}$. This share regression estimates the specified markup control function: it describes the determinants of wedges between prices and marginal costs, and is similar to the share regression in Gandhi, Navarro, and Rivers (2020), but adapted for cases of imperfect competition with unobserved prices.

Importantly, it also recovers an estimate of the error $\hat{\varepsilon}_{it}$, and therefore $\hat{b}_{it}$. Estimating $\hat{\varepsilon}_{it}$ is the primary function of the first stage of proxy variable estimators (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg et al., 2015). Estimating it here allows us to replace the physical productivity control function assumption of these models with a markup control function assumption. However, the share regression alone cannot separate the impact of markups from output elasticities, since it still contains the unknown f^X_{it}. We now return to the revenue production function to identify the physical elasticity f^X_{it}. Specifically, we rewrite the output from estimating the first-order condition to

[4] $\log f^X_{it} - \mu_{it} = \log f^X_{it} - \log(\frac{1}{1+p_q}) = \log f^X_{it} + \log(f^X_{it} p_q) = \log q^X_{it} + \log p^X_{it}$.

isolate revenue productivity

$$
\begin{aligned}
c_{it}x_{it} - r_{it} &= \hat{s}_{it} + \hat{b}_{it} - \hat{\varepsilon}_{it} && (11)\\
c_{it}x_{it} - f(x_{it}, k_{it}) - \nu_{it} - \hat{\varepsilon}_{it} &= \hat{s}_{it} + \hat{b}_{it} - \hat{\varepsilon}_{it} && (12)\\
\nu_{it} &= c_{it}x_{it} - f(x_{it}, k_{it}) - \hat{s}_{it} - \hat{b}_{it} && (13)
\end{aligned}
$$

And we express the revenue production function as

$$
r_{it} = f(x_{it}, k_{it}) + g(c_{it-1}x_{it-1} - f(x_{it-1}, k_{it-1}) - \hat{s}_{it-1} - \hat{b}_{it-1}) + \eta_{it} + \hat{\varepsilon}_{it} \quad (14)
$$

Hence, we can directly estimate production function parameters from this equations.

2.3 Identifying Returns to Scale and Markups

A well-known limitation in the production function estimation literature is that it is impossible to separately identifying the flexible input elasticity and the returns to scale applying the proxy method on common datasets (Gandhi et al., 2020). Essentially, absent additional source of identification, there exists a continuum of observationally equivalent production functions that satisfy the identification restrictions imposed in the proxy approach. Hence, returns to scale and markups are jointly identified but it is not possible to separate them. Observing serially correlated input price might solve this non-identification problem in specific cases, or imposing additional restrictions on the production function or productivity process.[5]

To avoid this non-identification issue, we follow the solution proposed by Flynn et al. (2019) which does not require additional data or assumptions on the evolution of the production function or productivity process. Specifically, they show that it is possible to estimate markups by setting the degree of returns to

[5] Gandhi et al. (2020) and Flynn et al. (2019) provide a detailed discussion on the precise conditions under which the production function is point identified in the presence of markups.

scale ex-ante. The intuition for their result is that by imposing the returns to scale, thus reducing the free output elasticities, the variation usually captured by the proxy method can be used to estimate exclusively the flexible input elasticity. Their findings show that this approach drastically reduces the bias resulting from the non-identification result by up to twenty times. Therefore, we assume that firm production function has constant returns to scale.

2.4 Estimation

Here we describe the steps to implement our suggested estimator.

1. Estimate the share regression to recover revenue elasticities and firms' expectations on productivity. In practice, regress the intermediates log cost share of revenues ($c_{it}x_{it} - r_{it}$) on log inputs and markup determinants. The latter can include firm and time fixed effects, along with other observed variables included in the vector $\mathbf{D}_{it}$.[6] Use the predicted residual, $\hat{\varepsilon}_{it}$, to form $\hat{b}_{it} = \log \hat{\mathbb{E}}[\exp(\hat{\varepsilon}_{it})]$, and therefore recover $\hat{s}_{it} = \log \widehat{f_{it}^X - \mu_{it}} = c_{it}x_{it} - r_{it} - \hat{b}_{it} + \hat{\varepsilon}_{it}$.

2. Specify functional forms for the production function, $f(x_{it}, k_{it})$, and the Markov process, $g(\nu_{it-1})$, such as Cobb-Douglas technology with AR(1) productivity, or translog technology with quadratic Markov productivity. Note that, as explained above, $f(x_{it}, k_{it})$ must satisfy a scale elasticity assumption, such as constant returns to scale. In practice, the main concern is how to model the intermediates elasticity, because this term directly affects the markup as seen from the first-order condition 4.

3. Combine the estimates $\hat{s}_{it}$, $\hat{b}_{it}$, and $\hat{\varepsilon}_{it}$ with data r_{it} and $c_{it}x_{it}$ and the specified functional forms for $f(x_{it}, k_{it})$ and $g(\nu_{it-1})$ to form the revenue productivity shock $\hat{\eta}_{it} = r_{it} - f(x_{it}, k_{it}) - g(c_{it-1}x_{it-1} - f(x_{it-1}, k_{it-1}) - \hat{s}_{it-1} - \hat{b}_{it-1}) - \hat{\varepsilon}_{it}$. This shock will be a function of the parameters in $f(x_{it}, k_{it})$ and $g(\nu_{it-1})$.

[6]Researchers can specify the information in $\mathbf{D}_{it}$ in line with their models. Examples include market shares, exporter dummies, location dummies, etc.

4. Finally, leverage the stochastic revenue productivity process to estimate the parameters of $f(x_{it}, k_{it})$ and $g(\nu_{it-1})$. Specifically, use the moment conditions formed by

$$\mathbb{E}[\hat{\eta}_{it}|k_{it}, \hat{s}_{it-1}] = 0$$

The specific moments depend upon the specifications of $f(x_{it}, k_{it})$ and $g(\nu_{it-1})$. For instance, we may add moments for a translog production function by including squares and interactions of the nonflexible inputs. We may also add lags and interactions of the revenue share vector, $\hat{s}_{it}$, to control for more complicated specifications of the revenue productivity process, $g(\nu_{it-1})$. For instance, we could use $\hat{s}_{it}^2$ to control for a second-order process. By construction, these additional moments are orthogonal to revenue productivity innovations. In economic terms, costs, demand, and conduct are determinants of markups, which co-evolve with the TFPR process. Innovations to this process are therefore orthogonal to any functions of the costs, demand, or conduct embedded in the $\hat{s}_{it}$ vector. This two-step approach can also be implemented as a single step GMM problem by jointly minimizing the residuals in the share regression together with the moment conditions from the second step (Wooldridge, 2009).

A final notes is in order. There must be independent variation in μ_{it} which does not enter $f(x_{it}, k_{it})$. This variation can come from the fixed effects ι_i and τ_t, or the $\mathbf{D}_{it}$ vector of other firm-time observables (we discuss specific examples in Section 3). With this variation, the model identifies physical quantity elasticities and markups. Without this variation, the nonparametric underidentification arguments of Gandhi et al. (2020) will apply. However, we view this requirement as minimal – the existing revenue productivity literature suggests persistent dispersion across firms that is independent of other latent variables such as physical productivity (Foster et al., 2008). Moreover, it is consistent with empirically successful models of firm dynamics, such as those studied in Sutton (1991).

Discussion The production function approach is a very powerful tool to estimate markup as it requires minimal assumptions on the competition environment and the demand system. However, while it offers a relatively straightforward markup formula, estimating it with traditional tools requires price information. Often a prohibitive requirement. Alternatively, more structure can substitute for the lack of prices. Klette and Griliches (1996) was the first to proposed a solution of this kind introducing a specific demand system. In fact, it adopts a monopolistic competition environment with a constant elasticity of substitution demand system. Markups are constant across firms and time and can be separately identified markups from production elasticities in this case. De Loecker (2011) expands this method introducing a product specific CES demand showing how to aggregate up to the firm-level. More recently, Gandhi et al. (2020) analyzing this issue proposes to adopt a CES demand system with time-varying elasticities to allow for variation over time. We see our proposed method as a generalization of the Klette and Griliches (1996)'s correction. In terms of additional assumptions, we propose a much more flexible environment that can nest a wide range of the most used models of firm dynamics and we allow markup to vary along different dimensions.

3 Comparison to Related Literature

In this section, we detail how our estimator compares to commonly used methods to estimate production functions and then illustrate how to apply it to models with imperfect competition.

3.1 Comparison to Existing Competitive Models

We first discuss the proxy variable class, then the dynamic panel class. Both methods form moments from information and timing assumptions about the productivity process, but differ in other assumptions to achieve identification.

3.1.1 Proxy Variable Estimators

The proxy model of production (Olley and Pakes, 1996; Levinsohn and Petrin, 2003; Ackerberg, Caves, and Frazer, 2015; Gandhi, Navarro, and Rivers, 2020) identifies production functions using assumptions about the unobservable state. Proxy models assume that productivity is Markovian and that productivity can be written as a control function of observables. These two assumptions allow one to use lagged inputs to control for current productivity, solving the transmission bias. More formally, the assumptions of the proxy model are:

Proxy Variable Assumption 1: Physical productivity follows a first-order Markov process: $\omega_{it} = g(\omega_{it-1}) + \eta_{it}$.

Proxy Variable Assumption 2: Physical productivity is a control function of observables: $\omega_{it} = m(x_{it}, k_{it})$.

Proxy Variable Assumption 2 ensures that past inputs can proxy for current productivity through the productivity process. Then substitution yields

$$\begin{aligned} q_{it} &= f(x_{it}, k_{it}) + \omega_{it} + \varepsilon_{it} \\ &= f(x_{it}, k_{it}) + g(\omega_{it-1}) + \eta_{it} + \varepsilon_{it} \\ &= f(x_{it}, k_{it}) + g(m(x_{it-1}, k_{it-1})) + \eta_{it} + \varepsilon_{it} \end{aligned}$$

Identification proceeds by forming moments with the composite error term $\eta_{it} + \varepsilon_{it}$.

With revenue data, this derivation becomes

$$\begin{aligned} r_{it} &= f(x_{it}, k_{it}) + p_{it} + \omega_{it} + \varepsilon_{it} \\ &= f(x_{it}, k_{it}) + p_{it} + g(\omega_{it-1}) + \eta_{it} + \varepsilon_{it} \\ &= f(x_{it}, k_{it}) + p_{it} + g(m(x_{it-1}, k_{it-1})) + \eta_{it} + \varepsilon_{it} \end{aligned}$$

The appearance of prices on the right-hand side is the origin of the omitted price bias terminology.

Our approach is a direct modification of these two assumptions. For the first, we assume revenue productivity (TFPR) follows a first-order Markov process, instead of physical productivity (TFPQ). This assumption is consistent with Foster, Haltiwanger, and Syverson (2008), which finds that revenue productivity exhibits similar levels of persistence as physical productivity. It is also implicit in existing work that estimates revenue production functions, or explicit in existing work that attempts to correct for the omitted price bias with richer data or stronger parametric structure (Klette and Griliches 1996, De Loecker 2011, De Loecker and Warzynski 2012). For the second, we assume markups are a control function of observables, instead of physical productivity. Although markups and productivity are both unobservable, researchers typically impose some markup-setting process or rule in modeling (for example, monopolistic competitors facing a constant price elasticity of substitution demand system). Therefore, we view our assumption on markups as less limiting. More generally, one can think of our model as a version of a proxy variable estimator, in which we are proxying for markups instead of proxying for productivity. It is built to estimate markups, and also allows us to relax some of the physical productivity assumptions.

3.1.2 Dynamic Panel Estimators

The dynamic panel approach pioneered by Blundell and Bond (2000) is a commonly used alternative to proxy variable approaches. Dynamic panel models maintain the same basic structure of production. However, they impose linearity on the productivity process.

Dynamic Panel Assumption: Physical productivity ω_{it} follows an AR(1) process: $\omega_{it} = \rho\omega_{it-1} + \eta_{it}$.

The unobserved term η_{it} is uncorrelated with all past and future input choices.

Then differencing the production function yields

$$
\begin{aligned}
q_{it} - \rho q_{it-1} &= f(x_{it}, k_{it}) - f(x_{it-1}, k_{it-1}) + \omega_{it} - \rho\omega_{it-1} + \varepsilon_{it} - \rho\varepsilon_{it-1} \\
q_{it} - \rho q_{it-1} &= f(x_{it}, k_{it}) - f(x_{it-1}, k_{it-1}) + \eta_{it} + \varepsilon_{it} - \rho\varepsilon_{it-1} \\
q_{it} &= \rho q_{it-1} + f(x_{it}, k_{it}) - f(x_{it-1}, k_{it-1}) + \eta_{it} + \varepsilon_{it} - \rho\varepsilon_{it-1}
\end{aligned}
$$

The appearance of lagged quantities on the right-hand side is the origin of the dynamic panel terminology. Identification proceeds by forming moments with the composite error term $\eta_{it} + \varepsilon_{it} - \rho\varepsilon_{it-1}$.

With revenue data, this derivation becomes

$$
\begin{aligned}
r_{it} - \rho r_{it-1} &= f(x_{it}, k_{it}) - \rho f(x_{it-1}, k_{it-1}) + p_{it} - \rho p_{it-1} + \omega_{it} - \rho\omega_{it-1} + \varepsilon_{it} - \rho\varepsilon_{it-1} \\
r_{it} - \rho r_{it-1} &= f(x_{it}, k_{it}) - \rho f(x_{it-1}, k_{it-1}) + p_{it} - \rho p_{it-1} + \eta_{it} + \varepsilon_{it} - \rho\varepsilon_{it-1} \\
r_{it} &= \rho r_{it-1} + f(x_{it}, k_{it}) - \rho f(x_{it-1}, k_{it-1}) + p_{it} - \rho p_{it-1} + \eta_{it} + \varepsilon_{it} - \rho\varepsilon_{it-1}
\end{aligned}
$$

Absent additional assumptions, we cannot proceed further without price data. However, combining this equation with a version of our earlier Markovian revenue productivity assumption ($\nu_{it} = g(\nu_{it-1}) + \eta_{it}$), we can make progress. Suppose our earlier assumption of Markovian revenue productivity holds, and further suppose it is linear, so that revenue productivity follows an AR(1) process: $\nu_{it} = \rho\nu_{it-1} + \eta_{it}$. We can then proceed as follows:

$$
\begin{aligned}
r_{it} - \rho r_{it-1} &= f(x_{it}, k_{it}) - \rho f(x_{it-1}, k_{it-1}) + p_{it} - \rho p_{it-1} + \omega_{it} - \rho\omega_{it-1} + \varepsilon_{it} - \rho\varepsilon_{it-1} \\
r_{it} - \rho r_{it-1} &= f(x_{it}, k_{it}) - \rho f(x_{it-1}, k_{it-1}) + \eta_{it} + \varepsilon_{it} - \rho\varepsilon_{it-1} \\
r_{it} &= \rho r_{it-1} + f(x_{it}, k_{it}) - \rho f(x_{it-1}, k_{it-1}) + \eta_{it} + \varepsilon_{it} - \rho\varepsilon_{it-1}
\end{aligned}
$$

This derivation does not require a markup control function, and therefore suggests a similar trade-off as in the competitive case: researchers may impose more structure (linear) on the productivity process to avoid assumptions that observables (inputs) and fixed effects span unobservables (markups).

3.2 Comparison to Imperfect Competition Models

Our solution generalizes much of the existing literature. In our earlier setup, we showed that our markup function identifies markups so long as $\mathcal{M}_{it}$ is determined partially independently from inputs. Now, we offer several commonly used parametric examples of markup functions.

3.2.1 Constant Markups: Monopolistic Competition and CES Demand

Suppose that firms are monopolistic competitors facing a constant price elasticity of substitution demand system. Suppose further that these firms compete in a number of industries j. In this environment, a firm i in industry j faces a demand curve given by $Q_{it} = Q_{jt}(\frac{P_{it}}{P_{jt}})^{-\sigma_j}$. Firm optimization implies that markups are constant within industries and given by $\mathcal{M}_{it} = \mathcal{M}_{jt} = \frac{\sigma_j}{\sigma_j - 1}$.

In the context of our markup function, assuming monopolistic competition with CES demand implies that $\mu_{it} = \mu_{jt}$: markups are fully determined by a constant within industry. One may recover markups and elasticities by simply including an industry fixed effect in the share regression.

This was originally noted in Klette and Griliches (1996). This paper shows that, in this case, one can estimate (industry-level) production functions by including controls for the industry quantity production. Klette and Griliches (1996) uses the estimating equation

$$r_{it} = \beta_0 + \frac{\sigma_j - 1}{\sigma_j}(\beta_X x_{it} + \beta_K k_{it}) - \frac{1}{\sigma_j} q_{jt} + \nu_{it}$$

Here, q_j is an industry-level price index which comes from the monopolistically competitive environment. Estimation can then proceed using (observed) industry-level output.

Gandhi et al. (2020) extends the Klette and Griliches (1996) model to allow for time-varying price elasticities of demand. We can easily accommodate such an extension by including a time fixed effect in the share regression.

In sum, our approach is a generalization of the Klette and Griliches (1996) correction that allows for more conduct and demand structures than monopolistic competition with CES demand. This generalization is especially important as it allows credible scaling of production-based markup estimators across many industries and time periods, where these earlier assumptions might be considerably off.

3.2.2 Variable Markups: Oligopolistic Competition and CES Demand

Suppose now that firms are oligopolistic competitors facing a variable price elasticity of substitution demand system. This is the case when demand is of the nested CES form: the final good is a CES demand of a continuum of sectors and each sector good is a CES aggregate of differentiated products (Atkeson and Burstein, 2008). In this setting, the elasticity of demand is a combination of the elasticity of substitution across sectors and within own sector, weighed by the market share of each firm.[7]

In Cournot competition, markups are a function of firm-specific demand elasticity: $\mu_{it} = \frac{\epsilon_{it}}{\epsilon_{it}+1}$, with the latter being

$$\epsilon_{it} = \frac{1}{\eta}(1 - s_{it}) + \frac{1}{\theta} s_{it} \tag{15}$$

with s_{it} being the market share of firm i, and θ and η the elasticities of substitution across and within sector, respectively.

In the context of our markup function, assuming oligopolistic competition with nested CES demand implies that markups are fully determined by a combination of two constants and firms' market shares. One may recover markups and output elasticities by controlling for fixed effects and industry market shares in the share regression.

[7] If firms compete in quantities, such combination takes the form of a harmonic mean. If firms compete in prices, it becomes a weighted mean.

3.2.3 Variable Markups: Firm-Time Characteristics

We explored two specific models of imperfect competition so far and showed how our method can recover markups from these cases. In general, our method is flexible enough to allow the researcher to adapt it to recover markups depending on the framework analyzed and the data available. If the researcher has data on firm-level characteristics which determine markups, then our model identifies market power and production elasticities by putting these into $\mathbf{D}_{it}$. For instance, advertising, managerial practices, research and development (Doraszelski and Jaumandreu, 2013), export status De Loecker and Warzynski (2012), or product mix De Loecker (2011) all might be associated with markups, and therefore added to the share regression. Any other observable market characteristics that vary by firm-time, such as combining geographic variation with consumer income variation, may also be added, depending on the researcher's model.

If revenue market shares determine markups, then in our model we have $\mathbf{D}_{it} = \frac{R_{it}}{R_{jt}}$, where R_{jt} is industry revenues. Researchers may define industries j in whatever way appropriate, such as common industry codes, or broadly or narrowly defined product markets. Then the markup control function may again be used to identify markups and output elasticities by adding market shares to each. Unlike models such as nested CES, this approach does not impose a parametric relationship between market shares and markups. Rather, the data determine the relationship.

In microfounding this control variable, typical models of competition such as homogenous product Cournot and differentiated product Bertrand result in a mapping from markups to quantity market shares, not revenue market shares. Of course, if we had quantity market shares, we would have quantity information, which would obviate the need for revenue data corrections.

4 Conclusions

This paper addresses the general issue of estimating markups with commonly available dataset by proposing a method to estimate markups with revenue data, without requiring information on prices. It recovers unbiased and consistent markup estimates and requires only common regression techniques and information available in most data settings. The method is based on a production function estimator that flexibly models markups as a specified function of observables and fixed effects and modifies physical productivity process assumptions into revenue productivity process assumptions. This way, we solve the omitted price bias without imposing additional assumptions on the demand side or the competition structure.

Modelling markups as a function of observable firm characteristics and fixed effects captures insights from recent macroeconomic and trade models featuring variable markups. Those controls indeed are meant to capture factors determining variable demand elasticities such as market complementarities or industry specific characteristics. In addition, assuming a stochastic process for revenue productivity rather than for quantity productivity, makes explicit assumptions underlying most of the recent work done in the literature investigating markups and is in line evidence on the dynamics of revenue productivity.

In conclusion, our proposed method provides a simple and effective way to estimate markups using only revenue data. The method has important implications for researchers and policymakers interested in understanding the dynamics of product markets and the impact of market power on economic outcomes. The proposed method has several potential applications and can be extended to other settings, making it a valuable tool for future research.

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